

Context Specific Evolutionary Design: An Analysis on Computational Abstraction of Modern Urban Complexity

Yufeng Zhai, Elisabeth Riederer

Architectural Association (AA) School of Architecture
London, United Kingdom
Yufeng.Zhai@aaschool.ac.uk, Riederer@aalumni.org

ABSTRACT

Evolutionary design is used to adapt urban systems to predictions such as rapid growing density and effects of climate change scenarios. These effects have weakened the strategies on which ancient cities were built and thrived. Fez el Bali's Medina can be seen as a drastic case and is therefore chosen for theoretical investigations. The Medina nowadays has lost its quality as a functioning system, characterized by a coherent relation of hierarchical order and randomness based on a cultural heritage.

In this paper's architectural approach a city is redeveloped on the basis of the earlier well-functioning ancient city after which an urban patch is then developed further. In order to react to the unpredictable changing conditions, we propose an open system generated by outlining the qualities the Medina was built on and developing this further to be able to react to changes within the city and beyond it.

The key element of this paper is to expand on the level of complexity in Evolutionary Design by operating on the urban scale and contextualized to push its computational potential. Expanding on the application of design strategies of Genetic Algorithms (GA) we incorporate rule-based multi-scale procedural modeling based on the vernacular urban qualities, while examine urban morphological variation evolved in response to conflicting criteria by means of a Multi-Objective Evolutionary Approach (MOEA).

Author Keywords

Evolutionary Design; Multi-Objective Genetic Algorithm; Urban design; Generative Design; Super Blocks; Morphological Variations.

1 INTRODUCTION

Evolutionary Computation to Encounter Urban Complexity

Examining the changes in biological structures has shown how the forms are created through invention and expansion

with existing body parts being crucial to innovation [1]. Evolutionary trends more frequently follow available paths so that major changes in evolution occur via mutations in existing genes [2]. The modern city as a complex organism shaped by its dynamics might relate to these evolutionary process to improve adaptation. Evolutionary strategies such as innovation building upon existing structures also seem to be found in well-adapted urban structures. Even though the methods of Evolutionary Computation such as a Multi-Objective Evolutionary Approach (MOEAs) may be seen as an appropriate tool in urban design and have been proven to be an efficient design method to encounter a certain level of complexity, Evolutionary Design is still rarely used in practice. This might be due to the fact that evolutionary models mostly aim to design scripts, which are faster and more effective. Rationalizing scripts might lead to a downside for the design outcome and to questioning the integrity of GA [3]. This development might be initiated by Evolutionary Design most commonly used as a form-finding tool to explore novel forms. These form-finding processes, lack of context, and therefore also practicing abstraction of context specific data as an input for GA. By examining this abstraction further we hope to expand on the topic of context specific Evolutionary Design and trigger its application in practice.

The key element of this paper is to expand on the level of complexity in Evolutionary Design by operating on the site-specific urban scale to push its computational potential. Addressing the application of Evolutionary Design and expanding on the complexity level of Evolutionary Computation as a design method we incorporate complex form generation rule sets and multi-scale procedural modeling while examining urban morphological and topological variations evolved in response to conflicting criteria by means of a MOEA.

The following studies examine the ability of a GA to encounter the degree of modern cities complexity in design solutions. Theoretical investigations are exposed through

redeveloping an urban center to generate optimized design solutions without compromising its cultural values, urban fabric and typology while addressing nowadays conflict of social structures. Due to tourism and overpopulation many ancient cities are affected by social conflicts with different stakeholders fighting for more space respectively. By studying these vernacular well-functioning self-organized urban regions and analyzing the contemporary urban issues within, we are aiming to redevelop and improve the “better equipped” urban fabrics.

Fez el Bali’s Medina to Demonstrate Urban Complexity

Many ancient cities around the world have survived centuries and are well adapted to their environmental context as well as the social economical context through self-organized evolutions. Evolutionary algorithms have the potential to generate the same level of complex well adapted urban design options with only a fraction of the time. This makes it an ideal and efficient design tool for redevelopment in historical urban regions.

One of the typical vernacular self-organized urban regions facing severe challenges nowadays is Fez el Bali (Medina of the city Fez, Morocco). Fez el Bali’s Medina builds upon a complexity characterized by a coherent relation of environmental factors and hierarchical order based on cultural heritage and is therefore chosen for investigations. In recent years, the living quality of Medina’s residents has been reduced due to the increasing population density and rapid growth in tourism. Therefore the Medina functions as an admonishment for city developers to consider future developments and to include these in their planning. Multiple statistical data indicates this development and leads to a growing conflict of different stakeholders. Local residents, small business owners providing services and merchandise to tourists and local craft workshops each seek for more space.

2 BACKGROUND

2.1 Fez el Bali’s Medina

Fez el Bali, as part of the Medina of Fez, is located in the north of Morocco. With a population size of 156,000. The Medina covers an area of 280 ha and has a population density (2002) of 710 inhabitants per hectare [4]. It is also one of the world's largest pedestrian-only urban areas.

The medina Fez is dictated by the typical vernacular courtyard housing blocks. This typology is usually a multi-storey adobe house with an inner courtyard for an extended family dwelling. The fortified exterior facade provides strict privacy. The resultant streets often-occurs narrow and over shaded by the high walls.

Social Context and Urban Development

Fez el Bali is characterized by its high social interaction caused by the limitation of traveling as a result of a free-car area and by its limited public space, forcing people to interact more [4].

A rise in population density caused dilapidated building conditions and a deterioration of living conditions which further led to people migrating from the Medinas. Simultaneously and because of still existing high social structures (relationship and support systems) a shift against this trend appeared and new residents settled [6].

Most of the Medinas have become urban cores of much larger urban agglomerations; In Fez, only 12% of the population lives inside the Medina (relatively high value compared with the Medinas of Casablanca and Tangier (only 2%)) [7]. An analysis of the housing issues predicts a rise in population and a rise of tourism that will lead to conflict between stakeholders, seeking space, and local residents striving for residential areas, small business owners providing commercial shops for tourists and manufacturers seeking space to expand their workshops.

2.2 Urban System Modeling

In order to establish the systematic and computational model for the urban generative design we relate to Farzaneh’s[24] systematic urban modeling scenarios. As Farzaneh summarized, systematic urban modeling could be categorized into three scenarios:

1. Block dominated;
2. Network dominated;
3. Co-evolved

In the first two scenarios, either the building blocks or the street networks dominate the overall layout of the urban plan while the other emerges as a resultant pattern. Computationally, these two scenarios are easier to be modeled digitally. Block dominated scenarios link the program, land use, density to dimensions and plot geometrical configurations, while the network dominated scenarios focus on the interrelations between the typological nodes and edges of the street network. In the third scenario, coevolved model, the organizational pattern of the street networks and building blocks reciprocate and mutually adapt and modify each other. Which suggests a feedback loop in the generative process.

Given the characteristics of Medina Fes, the computational experiments presented in this paper focused on the block dominated scenarios.

2.3 Evolutionary Computation Literature Review

Evolutionary computation employs analogies to the biological phenomena of evolution and selection to generate algorithms that are used to obtain computational solutions to problems in numerous fields. These computational methods are suitable for finding solutions to complex problems with multiple objectives that may conflict with each other. The methods generate a set of solutions, which satisfy all objectives without any objective dominating and can then be examined for trade-offs [9]. Because of its background the terminology used in evolutionary computation leans heavily on genetics. For instance, a candidate solution to a problem is referred to as an individual; the sum of the defining properties of the individual is its genome, whilst an individual property is referred to as a gene, the value of

which is an allele [10]. Coupled with optimization, the evolved solution set is a robust and powerful alternative to a single, preference-based approach.

The earliest attempt of utilizing biological evolution inspired by problem solving and optimization could be dated back to the pre-digital computing period 1930s, Sewell Wright's works [11].

A major advancement in the evolutionary computation development was only computationally achievable after the advent of the computers. Several scholars emerged between the 1960s and 1970s simultaneously theorized their computational methods of evolutionary computation.

Overall, much of the research and methods published at this stage focused on the single-objective evolutionary algorithms (SOEA). For instance, John H. Holland's on GA [12], Lawrence J. Fogel published a paper 'On the Organization of Intellect'[20] about evolutionary programming (EP), Rechenberg [13] and Schwefel's[14] on evolutionary strategies (ES).

The first effort towards MOEA design was led by David Schaffer in the 1980s through his published paper 'Multiple objective optimization with vector evaluated genetic algorithms' [14].

Among all the scholars' research, the most significant contribution to increase the efficiency of the MOEA was developed by David Goldberg who introduced Pareto front evaluation and non-dominant selection strategy into the MOEA. [15] By implementing his strategy, the algorithm was able to incrementally optimize the fitness according to each objective independently, 'spread its population out along the Pareto optimal tradeoff surface' and balance variations towards multi objective while the fitness value could still be incrementally optimized. [21]

The introduction of Elitism and Archive by Eckart Zitzler to MOEA diverged the computational evolution model from the biological evolution model. Zitzler argued that the non dominant individuals (local optimal) in the earlier generations, may still dominate across later generations. Therefore, the elite solutions could be archived directly to the next generation to compete with newer generation. [17] The application of his method can be found in the algorithm Zitzler developed: Strength-Pareto Evolutionary Algorithm, SPEA (1999) and SPEA-2 (2001) [18].

The current state of art of Genetic Algorithms incorporate the strategies mentioned above and have been tested on robust computing efficiency. Some of the most popular GA strategies are Kalyanmoy Deb's NSGA III; Knowles and Conre's Pareto-Archived Evolution Strategy (PAES) and Zitzler SPEA-2.

As GA algorithms typically generate large data sets, one of the key developments in recent evolutionary computation lies in the post-evaluation and statistical analytical tool development. This has made it possible for the general users

like architects to evaluate and validate the algorithm result. For this reason, the statistical analytics and visualization tools became essential in the big data analysis.

The recent advancement in machine learning also advanced data analysis, for instance, the utilization of K-mean clustering in Wallacei Analytics[19] providing the users insight on how the GA algorithms developed their novel evolution strategies.

In the field of architectural computation, prior works have demonstrated the problem solving capacity of the MOEA in architecture and urban design. Calixto and Celani [28] contributed the theoretical framework of the MOEA utilization in spatial layout design, while other architects delved into the real world architectural problem solving. For instance, the project by Benjamin et al. [25] on the office layout optimization explored quantitatively optimized the intangible objectives like productivity among other objectives. Schwartz et al. [26] novel approach addresses the life cycle carbon footprint and cost of building redevelopment through MOEA.

In terms of MOEA application on the urban design, Makki et al[27] and Farzaneh et al[16] speculative design experiments evinced MOEA potential, however the projects context were largely underplayed and super block modelling were simplified.

3 COMPUTATIONAL EXPERIMENT

3.1 Constraint and Goal

Inquiry to site context identified the constraints of the site:

1. The complex urban pattern of Fes el Bai is dictated by the vernacular housing blocks. It is sensible to adopt this vernacular typology and urban pattern in the speculative redevelopment of the region to meet the Moroccan family structure and social customs.

2. Density constraints: conflicts between living tourism and industry growth. One of the immediate challenges observed in the Fes el Bali is to find a solution which can achieve reasonably high density and balance stakeholders needs, while still maintaining relatively sufficient area of streets and open public spaces. The maximal floor area ratio (FAR) was chosen as a numerical fitness criteria.

3. topographical and climatic constraints: the sloped site marks 66.6 m elevational change, the appropriate building strategy was needed to address the sloped terrain. In terms of the climate, Fes is Hot-summer Mediterranean climate. self shading provided by the 'Deep canyon' streets and blocks mediates microclimate. However, the internal courtyard receives inadequate solar gain daylighting due to the buildings. For this reason, the environmental objective of the optimization was to optimize the direct sunlight gain from the south during the winter season.

Drawing upon the understanding of the site constraints and literature review of MOEA. The Research question could be

stated as: can MOEA be used as an efficient tool to generate contextualized, climatically and socially adapted urban design solutions for historical urban area redevelopment of Fes el Bali?

3.2 Workflow

To answer the question, two sets of design experiments were set up in a consecutive manner. First experiment focuses on housing block cluster (2x2) generative design with only singular typology considered: the collective courtyard house. Experiment 2 focused on the urban scale superblocks and street network generation. The objectives defined for this experiment were the numeric abstraction of the site context analysis mentioned in the previous chapter.

3.3 Methodology

Experiments presented within this article address the issue of generating urban morphological variation evolved in response to conflicting criteria through the modeling of urban form by means of a MOEA. The computation was driven by the SPEA-2 algorithms, through the plugin called Octopus developed by Vierlinger [22], within the Grasshopper visual scripting environment. The procedural modelling and fitness evaluation were also customized in the Grasshopper environment to streamline the generative process.

3.4 Experiment 1 - Single Block

For initial experiments, an 20x 20m urban patch which consists of 4 square blocks was used for the design experiment. Each block was evenly divided into 4 courtyard houses. Regarding vernacular family structure, social costume and climatic conditions indigenous courtyard houses are a proof of concept and are therefore preferred as the primary typology.

Genes

The genes (building variables) include the number of the building floor (1), street width (2) as well as courtyard ratio (3) and room divisions(4). 1, The lowest floor number 0 leads to the formation of open spaces; thus the more variants of semi-open courtyards could occur. 2, Based on two-storey buildings and street widths of ranges 0.5 to 6 m. 3, The courtyard ratio is controlled by a scaling factor of

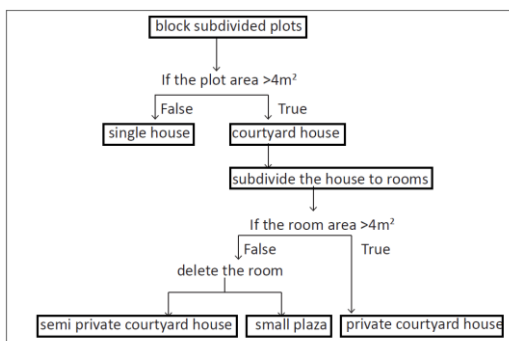


Figure 1. Rule based block generation - Boolean condition

building footprint in the range of 0.2-0.5. 4, 0m building height was also added for every single room to serve as a gene switch which can suppress the extruding building height function. This regulating gene could also lead to opening up the closed courtyard.

Fitness Objectives

1, maximize FAR. 2, maximize street area. 3, maximize south facing direct sunlight exposure (winter solstice 12pm solar vector)

Design Strategy

The ancient Urban qualities are mainly incorporated as ratios in the genes (street to building height and building footprint). We mostly address predictions of rapid growing density and their effects by means of our conflicting fitness criteria (maximizing FAR and solar exposure on the surface areas of courtyards and streets). Therefore Experiment 1 aims to encounter max. density, optimum solar radiation conditions for summer and winter as well as providing sufficient public spaces. The most important urban design considerations based on Fez el Bali's social and economic context analysis are social conflicts caused by overpopulation and tourism. FAR was chosen as the primary objective to be maximized to accommodate the stakeholders' need for space. To balance the real estate development revenue while ensuring sufficient urban public spaces and amenities we introduced the two conflicting objectives: maximizing FAR and maximizing street area. To counterbalance FAR we identified maximizing street area as the second objective.

In addition to the two objectives described above, the third environmentally driven objective, maximizing the courtyard solar exposure was defined based on the fact that the vernacular multi-level housing type and high-density neighborhood development often compromises the courtyard area, which significantly affects the indoor comfort including direct sunlight gain and daylighting. The population setup of the algorithm was 10 individual solutions per generation by a total of 100 generations. A Rule based procedural modelling was implemented to



Figure 2. Phenotypes and the fitness values

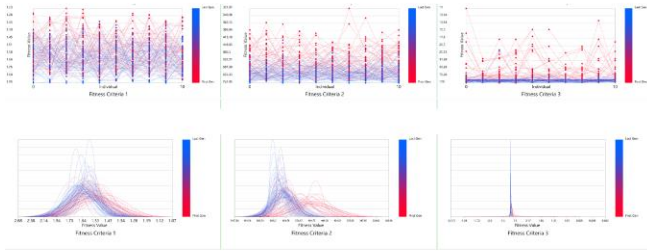


Figure 3. Top chart fitness values (0 marks fittest); bottom standard deviation graph (0 marks fittest)

ensure the algorithm generated architectural solutions are rational and sensible (see Figure 1).

Evaluation

The methods to evaluate the experiment were achieved by implementing a population-based fitness value analysis. By analyzing standard deviation of the individuals' fitness values across the generations provides more insight on the degree of variations and global trend of the optimization (see Figure 3). The standard distribution (SD) graph is generally a bell shaped curve. The wider the bell shape of the standard deviation, the more variations the generation contains). The centerline of each SD curve also indicates the mean fitness value of the generation. If the graph centerlines shift towards optimum direction from older to later generations, it can prove the objective being optimized. By overlaying SD curves across the generations, the exploitation/exploration weights could also be observed. Overall, experiment 1 was successful in terms of achieving sufficient variations while the 3 objectives were optimized incrementally. The graph also reflects clearly that more variations were generated to achieve objective 1 and 2, however, early convergence was observed in relation to objective 3, as the normal distribution curve narrowed down to the mean value in the early generations.

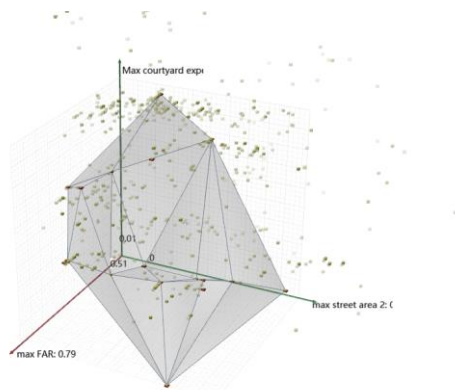


Figure 4. Objective space: solutions fitness value plotted in Cartesian coordinates.

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3.5 Experiment 2 - Urban Block in Context

Building on the design strategy, Experiment 2 aimed to generate an urban patch which addresses the issue of the land use and additional programs and building types. As the generative process of the whole urban design with multi-objectives is computationally heavy and prolonged, the generative process was divided into two hierarchical steps.

Step 1 – Global Scale Parcel Subdivision

In response to the topography, site, an orientation rule was introduced in the procedural modeling to address the topographic context. The main axis of the parcel always orients with the tangent vector of the nearest topographic contour to reduce elevational change within the building. In addition, Any parcel located on the steep (larger than 20%) terrain will be eliminated. In terms of parcel subdivision and scale variation, a quad-tree subdivision method was implemented. The subdivision logic applied subdivides the rectangular parcel iteratively into 4 parcels and coincides with context geometry. This process is repeated until the children parcels do not coincide with any context geometry (see figure 6). The merit of a quad-tree subdivision method lies in its efficiency to control the subdivision of the urban parcels as well as allowing scale variations. Compared to represent the context of the site, e.g. river, mosque and land use pattern, therefore only a single-objective was needed: maximize the total buildable area. The genes include total number of focal points, x axis spacing between the points, y axis spacing between the points. As a result of this strategy, smaller parcels are commonly preferred around the context while adapting to the geometric feature of the context. For instance, parcels follow the curvature of the meandering

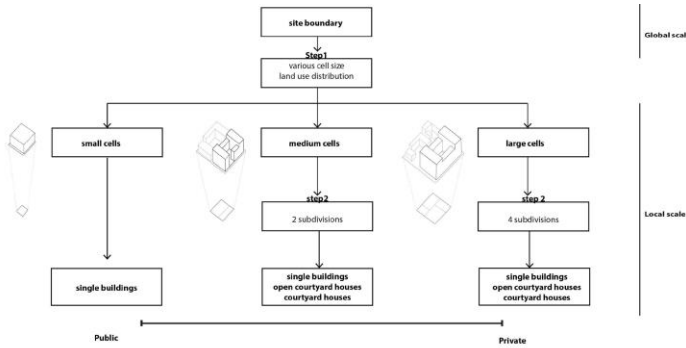


Figure 5. Experiment 2 workflow

canal. This also indicates a denser distribution at the focal points (see Figure 6).

Step 1 Results

The Genetic algorithm-generated result of 1000 solutions (10 individuals x 100 generations) converged into 4 distinctive clusters of solutions, the representative solutions from each cluster were closely scrutinized and compared. In each representative solution, one parcel type predominates. (see Figure 7) Although large parcel predominant solutions provide a larger building area, they perform less in terms of street connectivity and accessibility of each parcel. Regarding Medium parcel and small parcel predominant solutions provide a similar amount of building area. But small parcel solutions provide more open spaces, which serve as potential urban public spaces and can react to future urban growth.

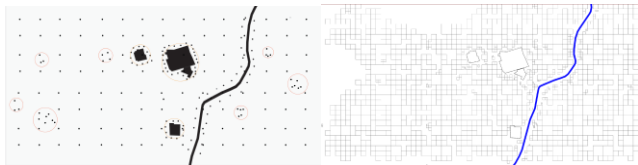


Figure 6. focal point distribution example and quad-tree subdivision

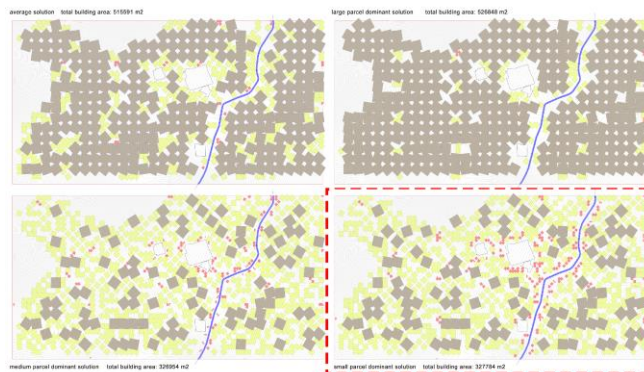


Figure 7. Step 1 GA results (top left: average solution,; top right: max larger parcel area; bottom left: max medium parcel area; bottom right: max small parcel area)

Step 2 – Superblock Generation

Following the urban parcel generative result in step 1, a selected region from step 1 was used as a base for step 2 due

to the limitation of computing power. The systematic modeling and algorithm setup were extracted and modified from experiment 1. Each parcel type was generated in parallel. Parcels were each divided into sub-blocks, subsequently creating secondary road networks within the sub-blocks. The differentiated modeling rule set was applied to the parcel based on the scale: 1, Small parcels were extruded directly as a single storey building, which serves as commercial use. 2, Medium parcels were subdivided to one workshop (single building) and one medium scale courtyard house. 3, Large parcels were subdivided into 4 large courtyard houses, while the subdivision also defined the secondary roads between the courtyard houses.

To control the building scale, several Boolean condition statements were imposed in the procedural modeling definition to delete the rooms, which are out of scale. In total, a full range of building variations were introduced, each corresponding to the three programs: commercial use, crafting production use and dwelling. The parcels variations consist of: Small single building-commercial use, Medium single building-crafting & production use, Medium courtyard house-single family dwelling, Large courtyard-collective dwelling. Objectives include: 1, maximize FAR. 2, maximize street area. 3, maximize south facing direct sunlight exposure (winter solstice 12pm solar vector)

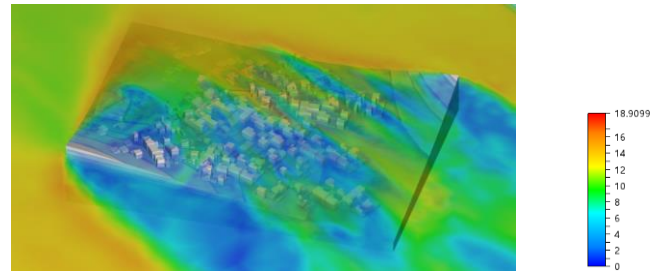


Figure 8. CFD result: air flow velocity and CFD boundary condition setup: inlet air velocity(m/s)

we examined the generated solution and performed Computational Fluid Dynamics analysis (CFD). CFD wind tunnels air inlet set up used prevailing wind direction (north west) and a proximate vertical logarithmic velocity profile to match the elevational change in real urban environment. The simulation result demonstrated clearly that the cross ventilation funnel. Two to three-story building height was in favor to minimize blockage and the stagnant air. It is also worth mentioning that the emerging larger open spaces enhanced air flow. the Building Solar gain can be indirectly measured by the solar exposure hours analysis which measures the number of hours the building can be directly exposed to the direct solar radiation. Both winter solstice and summer solstice day were analyzed to compare the urban layout and building massing configuration in relation to the solar angle. The solar study proved the dimension of the streets and courtyards resulted in adequate shaded outdoor space, which met the initial goal. However, the scale and dimension of the plazas and

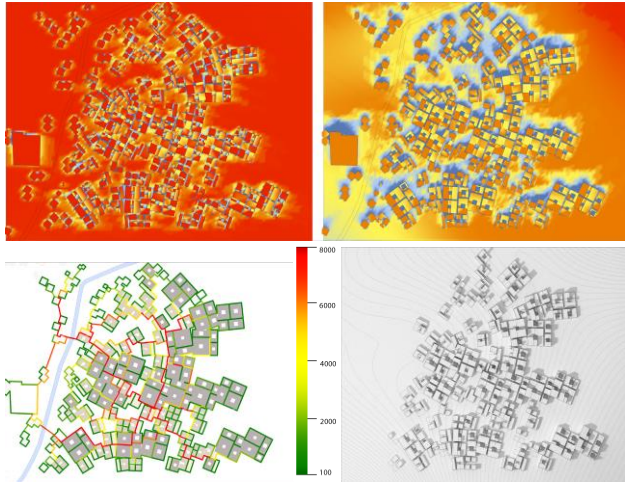


Figure 9. solar exposure hours analysis (unit: h) left: summer solstice right: winter solstice. Street network analysis (Betweenness centrality), building privacy mapping(light gray public, dark gray private)

the building configuration around them still need to be further articulated to prevent overheating.

3.1 Spatial Quality and Accessibility

As previously mentioned, the urban system generative model presented in this paper is predominantly driven by the urban superblock. As previously mentioned, the urban system generative model presented in this paper is predominantly driven by the urban superblock. To quantitatively examine the accessibility of emerging street networks, the network centrality (betweenness) analysis was conducted. From topology point of view, the street network could be abstracted as edges (street segments) and nodes(crossroads). Among all the possible shortest paths between any two nodes, 'The edge betweenness centrality is defined as the number of the shortest paths that go through an edge in a graph or network' [23] it reflects the street segments' popularity ranked by passengers and its potential to attract commercial activities. The analysis result was then superimposed with building privacy mapping. The privacy mapping took into consideration the program as well as the geometric feature on the two assumptions: 1.commercial and workshop buildings are more open compared to housing while 2. a semi-closed courtyard housing is less private than the closed courtyard house. The betweenness centrality analysis indicated the most frequently accessed streets (red) correlate to the distribution of commercial programs and smaller block subdivision. Emerging quality of the street networks exhibit similarity with the existing street pattern of Fes el Bali.

4 CONCLUSION

The computational experiments were successful in the sense that the numeric fitness values were optimized while morphologically, there were still a relatively large amount of variations. The building scale generations also covers the emergence of complex street networks and open spaces. These can be studied further. The demonstrated workflow method and approach showcase that the emergence of complex urban layout could be achieved by the bottom-up generative process. but the critical part is the rules setup, imposed control and Hierarchical order. Although the process and final solution are site-specific, the methodology and workflow could be potentially applied to a much wider variety of urban regions.

LIMITATIONS AND FURTHER STUDIES

The limitations of the process described in this paper were block dominated. The generation of the street network and the adaptations to the topography could be seen as an emergent result from the MOEA. This implies less control of street dimensions and network-hierarchy. Further research could potentially incorporate the street connectivity as a potential fitness objective to direct the MOEA towards a co-evolved urban scenario. Hereby the graph analysis in the computational model also informs the program distribution in the land use pattern as well as the block subdivision. In terms of environmental objectives, only the solar gain factor was considered in the generative process which is far from representing the reality of an urban microclimate. This suggests future research on the integration of additional environmental evaluation of wind criteria, in order to accurately evaluate the adaption of the solutions to the environment.

This design experiment would provide a valuable approach for high-density urban tissues, making use of vernacular topology, street pattern efficiency as well as applying MOEA. The presented strategy aims to prevent build cases such as the modern urban structure in Fes el Bali, deriving from the French colonization, called la Nouvelle Ville. La Nouvelle Ville shows poor adaptation to its environmental conditions. Comparing the two different urban systems in Fes el Bali, the modern urban structure Nouvelle Ville shows no relations to the Medinas urban strategies. In order to adapt urban systems to the complexity of a modern city of rapid growing density and effects of climate change scenarios incorporating vernacular building strategies as well as making use of MOEA were found as an appropriate approach and could be studied further.

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