ABSTRACT
This paper discusses a type of graph called “homeomorphically irreducible tree” (HIT) and its applicability for a formal study of symmetry in building plans. As a theoretical introduction, the mathematical properties of HITs are introduced through different historical building samples all of which display symmetry, proportion and homologous wings in their formal organization. The extracted principles are used to formulate a generative algorithm that reduces graph complexity to simple sequential numeric representation. This method is converted to a “homeomorphic machine” that is explored through generative plans. The aim of the paper is to introduce a new graph-based approach for potential morphological research into architectural symmetry.

Author Keywords
Homeomorphism, Graph, Digital Heritage, Symmetry, Morphology

1 INTRODUCTION
In the 20th century, the advent of computation transformed architectural research drastically by opening up the possibility of incorporating new practical methods with an influx of theoretical knowledge from other disciplines into architecture. Architects started using generative algorithms developed within computer science such as cellular automata, swarm simulations and various form optimization methods to develop new digital design tools, giving a more divergent and open-ended trajectory for the post-digital era [1]. Within this development, one can argue that design and research have become highly individualized and specialized, where core architectural principles were replaced with dynamic computational workflows that are justified by various performative, aesthetic or formal criteria. Before this contemporary development, the previous century also gave birth to various “schools of thought,” each proposing an alternative methodology and theoretical insight for the incorporation of computation into architectural research. Among these, shape grammars suggest the formulation of shape and generative rules proposing a transformational formalism that is applied to painting, sculpture, decoration, ornament and architecture [2]. Another avenue is called the space syntax, often focusing on the analysis of the interior spatial configuration of architectural plans by the perceptive, locomotive or behavioural aspects of humans [3]. Compared to grammars, the syntactic approach relies on discovering patterns among computationally analyzed spatial configurations that were represented as axial maps or graphs of connected lines [4]. The third formulation for architecture and computation was architectural morphology that gave primary consideration to the formal organization and physical properties of buildings to develop abstract mathematical models for the spatial and performative analysis of various building types [5]. One of the strengths of this approach is its interdisciplinary agenda, often combining knowledge from biology, computation and mathematics to establish an integrated scientific approach towards the geometric study of architectural form. This places architectural morphology closer to other disciplines in natural sciences due to its systematic approach and kinship to the utility of classification methods, while shape grammars have recently shown powerful computational permutations with “grammar machines” showing applicability with ongoing research in computer science [6].

One of the singularities that still await architectural research is its potential transformation with the advent of artificial intelligence and machine learning methods [7]. This places great concern on the formulation of analytical and computational methods that will be interpreted, modified and applied by neural networks. Current GAN (Generative Adversarial Network) models offer reinforced training methods where an array of building samples can be supplied to a machine learning algorithm to generate new building footprints, stylistic representations and furniture layouts [8]. In this example, a generic building outline informs how an optimal inner spatial division of an asymmetrical shape can be achieved as a top-down and subdivision based process showing great potential for future development.

The following paper aims to contribute to the ongoing investigation on computational morphology by presenting an alternative study combining symmetry and contracted graphs that are studied through historical works of architecture. Compared to other stochastic methods that determine spatial division from an outer boundary, this method is primarily driven by intrinsic generative principles...
centered on the notion of symmetry and branching morphogenesis in architecture. The aim of the research is two-fold. Firstly, formal computation is considered within a historical continuum where it can be applied to a broader class of historical works of architecture, potentially drawing links between them. This also requires a re-evaluation of core architectural principles such as symmetry and proportion that can potentially remedy architecture’s relationship with natural sciences. Secondly, computational methods need to be formulated primarily according to architectural principles that can overlap with various mathematical and algorithmic applications while conforming to the historical development of architectural knowledge. This aspect can have both theoretical and practical implications to architectural research and influence the formulation of new methods for machine learning in the future.

In this paper, a mathematical term “homeomorphism” is explored through contracted graphs that show potential application into the geometric study of heritage, while offering an alternative rule-set comparable to shape grammars and space syntax. Alternatively, this method will be defined as “formal computation” that neither requires a description of primitive shapes nor axial lines, in contrast, building forms are defined through abstract generative operations that recursively increase formal complexity of a geometric boundary while inner spatial divisions are simultaneously determined. The current application is directed to branching morphologies in architecture to introduce the method using simple numeric representation, while the overall geometric representation of building forms are kept abstract to avoid stylistic representations that could be added in future studies.

2 GRAPHS IN ARCHITECTURAL RESEARCH

Historically, graphs have been used in many other scientific fields and architecture due to their abstract nature and ability to capture complex structures with simple notations [9]. Among many mathematical models that are used for the study of architectural form, graphs most commonly found utility in highlighting connectivity and complexity of plans. In its mathematical definition, a graph is a collection of vertices connected with lines that can have diverse topological properties and classifications [9]. For instance, a graph with branching nodes with no edges joining a vertex to itself in a loop is called a tree, whereas graphs with cycles define more networked configurations or structures, like a mesh. In architecture, both graph types have been used to study various formal and spatial aspects of plans, such as the topological complexity of skeletonized alphabet plans [10], spatial connectivity and adjacency within rooms of a building [11], and enumeration of networking plan configurations [12]. In these examples, space is often abstracted using nodes for rooms, or the axis for corridors to reveal hierarchical or connectivity structure. Due to this depiction, graphs have been utilized to highlight the organization in continuous structures, particularly plans.

The motivation for the following work is the potential juxtaposition of hierarchical contracted graphs with architectural symmetry that can offer a new method for the morphological and computational study of building plans. Due to the presence of symmetry in these graphs, the primary case studies are selected from historical buildings that exhibit symmetry, patterned wing development and internal divisibility in plans [10]. A first glance at historical works shows that these properties are mostly found among institutional building typologies such as pavilion hospitals, asylums, prisons, military structures and schools. The broader analysis also shows HIT properties among many stylistic examples that exhibit wings. These can be found in modern, Gothic, Baroque and Renaissance architecture that also display symmetry in form and repetitive inner subdivision of rooms that remain secondary to the overall building form.

3 HOMEOMORPHICALLY IRREDUCIBLE TREES (HITS)

In Gus Van Sant’s 1997 film “Good Will Hunting” an MIT Professor in mathematics challenges his students with difficult math problems that take researchers years to prove [13]. To his surprise, these problems are not solved by any of his elite students, but by an anonymous mathematical prodigy who works as a janitor cleaning the hallways of the classrooms. One of the problems presented in the movie explores the possible connectivity graphs of trees of order ten, of which there are ten possibilities. This mathematical problem is more accurately defined as “homeomorphically irreducible trees” (HITs) that represent unique connectivity graphs described by points and lines [14,15].

A general characteristic of these trees is their topological properties that are related to symmetry and transformation. Firstly, a homeomorphic tree does not produce any cycles connecting nodes back to itself, forbidding triangles or other closed loops emerging inside the network (Fig.1). Secondly, a homeomorphic tree represents all possible topological variations of a unique node configuration. Reflection, rotation or scaling of nodes or trees do not produce another unique tree, as homeomorphism represents node connectivity of a certain mathematical space [16], because changing angles or lengths of a node does not change the topology or connectivity of the tree [14]. Finally, homeomorphic trees appear as contracted graphs.
where each node has at least three connections to other nodes, rendering nodes with two connections as reducible [15] (Fig.1).

In mathematics, HITs are characterized by single numbers – the total number of nodes in the graph. However, as the number of vertices increases, the unique graphs increase exponentially, making the identification and classification of individual trees difficult. This issue also presents problems for the computation of certain trees, as supplying single numbers lacks internal structure and hierarchy for the graph visualization. A solution to this issue can be mediated by differentiating between nodes of a tree, as internal and external nodes [15]. An internal node (white) inside the graph has at least three connections that can be to another internal or external node, while an external node has only one connection that is to an internal node. Due to the property of irreducibility of homeomorphism, HIT gives the option to enumerate the sequential connectivity of internal nodes in the simplified format of \{i_1, i_2, i_3, ..., i_j\} [15]. This model captures the valence of internal nodes as a sequence that gives a way of identifying each individual tree as well as offers a way of computing and comparing HITs. With this reformulation the unique trees on the figure 2 can now be redefined in the format of \{9\}, \{3,3,3,3\}, \{3*,3\}, \{4,6\}, \{5,5\}, \{4,3,4\}, \{3,5,3\}, \{3,4,4\}, \{3,7\}, \{3,3,5\} respectively, all of which have a total of ten nodes. This way the numeric representation not only captures the amount of internal and external connections and propagation of the network but also can say a lot about the overall architecture of the graph using simple sequential numeric parameters.

Using figure 2 some of the key visual and geometric properties of the HITs can be discussed. Firstly, HITs with non-constant degree values display a state of equilibrium between nodes while mediating radial and bilateral symmetries (i.e. \{4,6\}, \{3,4,4\}, \{3,7\}). Another aspect of these trees is the emerging axial propagation that is terminated with radial symmetries. In these trees, the sequential notation becomes more intuitive where the linear progression of the connected internal nodes represents the overall topology of the three. This property also renders notation for identical HITs such as \{4,6\}, \{6,4\} as equivalent and signifying both of them becomes redundant.

With the SAS model, the visual representation of constant degree value HITs presents some special cases. For instance, the tree \{3, 3, 3, 3\} for n=10 is represented in two different ways shown in figure 2 since sequential code of the HITs do not signify the internal connectivity nor propagation of nodes [15]. To differentiate them, the radially symmetric variation of \{3, 3, 3, 3\} is further simplified into \{3*, 3\} by attributing the second number of branches to all primary branches (See part 7 for further explanation of the * symbol). There are similar cases for \{4,4,4,4\} and \{3,3,3,3,3,3\} in n = 14, where the latter shows at least three different visual manifestations. Although the SAS model has limitations to represent radial trees as starting from central nodes lacks directionality, it offers robustness for axial HITs that can be numerically reduced to a sequence.

4 HOMEOMORPHIC ARCHITECTURE

In architectural research, graph theory has become a major tool to study various aspects of plan configuration, adjacency and connectivity of rooms, internal circulation of spaces [10, 11, 12]. In these studies, graphs often reveal hierarchical structures as well as formal relationships between homologous structures. For instance, graphs can be both applied to understand relationships on the micro scale-between rooms, or on the macro scale between wings that are connected with circulation. The former category often generates cycles due to multiple connections between adjacent rooms violating the acyclic nature of HITs. In the latter, branching morphologies that are made of homologous wings are suitable for HIT representation. Due to this morphological characteristic, the primary examples of homeomorphism in architecture are found among historical examples that exhibit symmetry in their plan. Furthermore, typologies with patterned wing development exhibit cases of homeomorphism that in most cases overlap with the building form. These characteristics are mostly found among hospitals, asylums, prisons, military barracks and schools that are often formed with identical wings or blocks (Fig.3). For instance, in radial prisons, such as Pentonville (1844), the wings containing cells reveal homeomorphism of the plan that overlaps with the symmetry axis of doubly-loaded corridors [17]. In asylums and pavilion hospitals, homeomorphism is mostly articulated with patterned wing development or repetition of the pavilion along an axis [10, 17]. In the latter, the “telephone-pole” plan allows the hospital to be easily expanded while offering an organic growth model to increase the capacity towards the open suburban landscape when needed [10].

Further evaluation of historical plans also shows the presence of homeomorphism among stylistic examples found in modern, Gothic, Baroque and Renaissance architecture that also display symmetry in form (Fig.3). In Gothic architecture, homology among wings or building partitions can be observed with repeating bays that exhibit identical vaults. In Salisbury, the main axis of the cathedral branch into multiple transepts while internally dividing these wings to similar vaulting structures. Homeomorphism can also be observed among Palladian villas with the bilaterally symmetrical house branching towards the
landscape. In these structures, symmetry mainly overlaps with internal corridors while each wing undergoes differentiation in the internal division of rooms [19]. In modern buildings such as hotels and offices, homeomorphism results in more repetitive wing plans that are joined at vertical circulation cores.

In figure 3, some of the analyzed historical examples are shown. These buildings are chosen based on the simplicity of their plan and application of homeomorphic principles to exemplify the approach. In this study, each building plan is first traced to reveal the internal skeleton of the building that generally overlaps with the circulation. Then points of convergence are found by intersecting the symmetry axis that defines the internal nodes of the graph. In the final step, two versions of the HIT diagram are shown. Compared to axial lines in space syntax [4], these graphs do not overlap with the longest visibility lines in space, but instead, define the symmetry skeleton of a building form. Shown under the “symmetry/form” column in Fig.3, the skeleton or symmetry axis of the building form is traced to reveal how it overlaps with the HIT graph. In the subsequent “HIT diagram,” the normalized version of the graph is shown to reveal patterned behaviour and repetitive wings of the plan. Among the analyzed samples, Pentonville exhibits a HIT diagram of {5} while Kirkbride Asylum plan shows patterned development in the form of {4,4,4...4} starting from the central block. Salisbury Cathedral can be

![Diagram](image-url)

**Figure 3.** HIT plan analysis of various historical samples.
simplified into \{3,4,4\}, while Hotel Nacional de Cuba (\{4,4\}), Herbert Hospital (\{4,4,4\...,4,3,3\}) and Cadillac Palace (\{4,4,4\...,4\}) show similar branching morphologies along an axis. This method shows the macro scale applicability of homeomorphic graphs while the sample sizes in each group can be increased to show more in-depth morphological similarities.

5 COMPUTATION OF HOMEOMORPHIC PLANS

The studied historical examples in the previous part show that in many historical works of architecture the symmetry axis coincides with the homeomorphic inner circulation connecting internal spaces or rooms within blocks. This approach gives a primary consideration for the overall form of the plan particularly expressing a differentiated and highly articulated outer geometrical boundary that separates internal spaces from the external environment. In the following model, this emphasis is transformed into an abstract formal algorithm to consider symmetrical relations in form while issues of scale and measurement in architecture are ignored to visualize the potential computational applications and simplicity of the HIT approach.

To investigate architectural symmetry computationally a simple algorithm structured around the mathematical notion of HIT is developed that can generate typical building plans with axial doubly loaded internal corridors. This recursive algorithm is developed in Python using the principles highlighted in the SAS model where the formal complexity of a tree graph can be represented with a sequence of numbers [15]. In figures 4-5 the procedural development of this algorithm is visualized for the HIT of \{3, 4, 6\} containing 12 nodes. Starting from the first internal node that acts as the root of the tree, the algorithm uses numeric code to propagate the tree structure in a linear fashion producing a major symmetry axis.

Figure 4. Generative modelling progression of \{3, 4, 6\}. Internal nodes are marked with white while external nodes are marked with black dots. Each step (numbers in HIT) shows the addition of external branches to achieve the total number of connections to an internal node.

Computation of homeomorphic plans occurs in two stages. In the first stage that is additive, the valency values of internal node sequence are used to generate a tree recursively (Fig. 4). For HIT of \{3, 4, 6\}, the first root node \{3\} is radially produced in the first step. Then, three new branches are added to produce the second internal node containing a total of four connections \{4\}. In the third step, five new branches are added for \{6\} that terminates the additive stage.

To turn the HIT diagram into a plan, the networking graph lines are offset while forming closed polygons located at the internal nodes (Fig. 5). For three and four-legged nodes, the thickening results in uniform polygons. In higher valency nodes, such as five-legged nodes, due to the uniform thickening of the wings, certain asymmetrical polygons start occurring in the plan. For the wings, the circulation is placed along the symmetry axis marked by the HIT graph of the form. This way each wing is composed of two symmetrical halves that are defined by an internal corridor and external-facing spaces or rooms.

Figure 5. Thickening and subdivision of HIT. Each axis is first offset into a polygon that is divided into three zones – circulation (grey), services (black) and spaces (white), the latter can be further subdivided into rooms using parametric thresholds.

Figure 6. Recursive inner subdivision of wings that determine spaces.

Once the form of the plan is thickened, each emerging rectangular polygon of the wing is recursively subdivided into two halves using a division threshold, progressively producing smaller spaces or rooms (Fig.6). This subtractive method takes inspiration from cellular division and packing of cells by developing symmetrical relationships among parts of the plan [20]. All the topological properties and recursive stages of the HIT architecture are controlled by parameters that allow for various configurations and generative studies of emergent forms.

6 ANALYSIS OF HOMEOMORPHIC PLANS

In the next phase, the HIT rules are used to produce permutations of various architectural plans using the existing mathematical codes for 4, 5, 6, 7, 8, 9 and 10 node trees (Fig. 7). To achieve comparable results, the algorithm is supplied with the same parameters for branch thicknesses and lengths, corridor to floor proportion and room subdivision coefficients. The resulting plan forms are analyzed according to their floor area, the number of spaces (white) created, corridor (grey) and service spaces (black), spaces with natural (white) and artificial (dark – grey and black) lighting / ventilation (Table 1). To compare the performative aspects of emerging trees, the ratios of various spaces are used.
The results show that among the produced HIT plans, radial symmetry causes an increase in service areas (Black spaces). As the number of nodes increases, the total outline of the plan form increases allowing for more natural lighting (White spaces). However, the graph variations for the same node count does not play any major role in the number of total rooms created or the overall floor area, except the cases with radial symmetry that produce connections to at least five external nodes. All the possible trees for 5, 6, 7, 8, 9 nodes produce similar number of spaces (rooms), and most of the trees produce similar circulation area / total area ratios, showing that adding new branches does not change the spatial performance of the plans. However, adding more branches often increases the outer surface area for the same node count trees and can improve daylight or ventilation performance. This becomes evident when constant value HITs (\{5\}, \{6\}, \{7\}, \{8\}, \{9\}) are compared to the similar node count trees.

In figure 8, the ratios for BT (Black Areas / Total Areas) and DL (Black Areas / White Areas) are plotted in a graph for the analyzed HITs. The values for single node HITs are connected using a line to show the standard distribution for daylight/ventilation performance. This graph shows that as the inner node count increases, the HIT moves to a lower BT (reducing service areas) or DL (increasing external-facing spaces) gaining better performance for natural light or ventilation (Fig.8). Furthermore, multiple inner node HITs with three and four-legged networks generate a higher performance (Lower BW) compared to five or more legged nodes and single number HITs.

Table 1. Topological comparison of HITs for node counts between 4 and 10. TN: Total Nodes, IN: Internal Nodes, EN: External Nodes, FB: Form Boundary / Branch Length, S: Number of Spaces, CT: Circulation Area / Total Area, GT: Grey Areas / Total Area, BT: Black Areas / Total Area, BW: Black Areas / White Areas.

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Figure 7. HIT plans showing n = 4 (\{1\}), n = 5 (\{4\}), n = 6 (\{5\}, \{3,3\}), n = 7 (\{6\}, \{3,4\}), n = 8 (\{7\}, \{3,5\}, \{4,4\}, \{3,3,3\}), n = 9 (\{8\}, \{3,6\}, \{4,5\}, \{3,3,4\}, \{3,4,3\}), n = 10 (\{9\}, \{3,7\}, \{4,6\}, \{5,5\}, \{3,3,5\}, \{3,4,4\}, \{3,5,3\}, \{4,3,4\}, \{3,3,3,3\}).

Figure 8. Daylight and natural ventilation performance of analyzed HITs. A dotted line connects single number HITs (White Dots). HITs with more internal nodes move to lower BT and DL areas improving performance.
7 HOMEOMORPHIC PLAN VARIATIONS
In the last step, a “HIT machine” is created to show a generative application for architectural plans. As an improvement, the sequential numeric HIT code is arranged with an additional symmetry condition, where an internal node number can be marked (*) to apply the same HIT progression to all the subsequent nodes to generate diverse building forms with local and global symmetries. This way, the form of a building can be represented with numbers, like a compressed DNA sequence, offering various genetic studies and comparisons. In figure 9, homeomorphic form variations of the improved algorithm are shown that can be further elaborated with additional geometric rules.

8 CONCLUSION
This paper discussed the mathematical term “homeomorphically irreducible tree” (HIT) to offer a simplified graph approach for the study of architectural plans. The presented method offers a simplified sequential numeric code for branching architectural morphologies in the form of \( \{i_1, i_2, i_3, \ldots, i_j\} \) that can generate wide parametric variability. The results show that HIT algorithm can be both used to study the notion of symmetry among various historical building types and as a generative modelling tool. However, HITs cannot be used to represent plans with courtyards or other circulation systems with circuits as cycles are forbidden in their configuration. While the analyzed historical examples show homeomorphic properties, further investigation into the directionality and symmetry properties of HIT is required to offer a comprehensive analysis of each historical building type. To summarize, this paper presented an alternative use of graphs for architectural research that is described through “formal computation.” With this perspective, building plans are primarily studied through their outer form boundary and symmetry axis while inner spatial division can be achieved by subdividing each branch or wing of a building. This way, HITs offer a higher level of abstraction and parametric simplification to generate complex building forms while providing an exciting computational avenue for a genetic study of architectural morphology.

AUTHORSHIP INFORMATION
This paper is a continuation of a journal paper titled “Homeomorphic Architecture,” accepted to be published for Environment and Planning B in March 2020. The work presented here expands on the idea by showing analysis of other building types, providing performative comparisons of variations and generating homeomorphic plans.

REFERENCES
8. A recent promising example to this study is the ArchiGAN that was developed by Stanislas Chaillou as a Harvard GSD master’s thesis project.